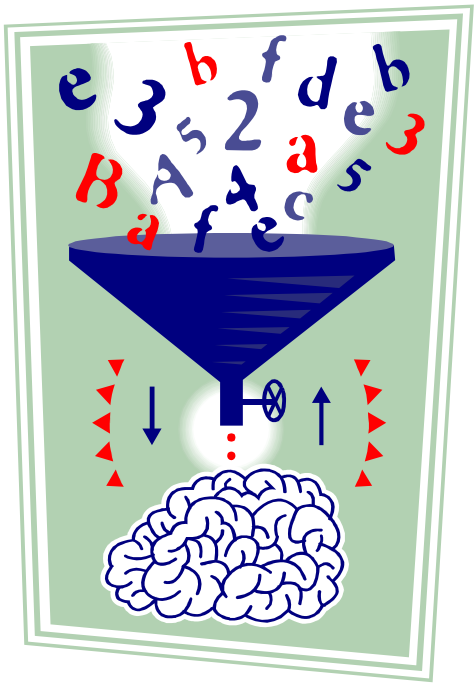
2012-2013 Algebra Academy

Exploring Student's Mathematical Thinking

Probing the Math Needed for Algebra

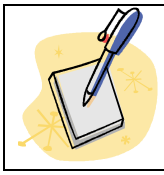
for Special Education Staff of

District 287 & Member Districts



Wednesday, Sept. 19 (Room 321) **EQUALITY**
Wednesday, Nov. 7 (Room 321) **MODELING WORD PROBLEMS**
Wednesday, Dec. 12 (Room 321) **RELATIONAL THINKING**
Wednesday, Jan. 16 (Room 321) **OPERATIONS & BASIC FACTS**
Wednesday, Feb. 27 (Room 321) **FRACTIONS AND DECIMALS**
Wednesday, May 1 (Room 321) **EFFORT, PROBLEM SOLVING & REASONING**

Facilitators: Nancy Nutting nancynutting@comcast.net
Mary Peters mkpeters@district287.org
Christina Shidla cashidla@district287.org
Scott Swanson saswanson@district287.org



Make a Note . . .

Program Facilitor for Professional Learning
Jennifer Nelson jlnelson@district287.org 763-550-7241

287 Mathematics Website – Algebra Academy
Check for session handouts, assessments, templates & other resources
http://courses.district287.org/mod/page/view.php?id=10229

Minnesota STEM Teacher Center Frameworks
http://www.scimathmn.org/stemtc/frameworks

AGENDA	INSTRUCTIONAL STRATEGIES
Spill the Beans PLC Time – Looking at the Equality Summative Assessment Focus on Addition & Subtraction Word Problems: • Sorting problems • Problem types • Bar modeling Working with the Vocabulary of Mathematics – MCAs+ LUNCH Focus on Multiplication/Division & Ratios and Rates Word Problems • Sorting problems • Problem types • Bar modeling Working with the Vocabulary of Mathematics – MCAs+ Carousel of Problems Student Assessments Overview PLC Planning Time Feedback	Unlocking Information in a Word Problem (p. 7) "Just Right" Problems (p. 8) Problem of the Week (p. 8) Developing Relational Thinking through related problems (p. 8) BAR MODELING (p. 10 +) Strategies for learning mathematical vocabulary (separate handout)

Comparing Summative and Baseline Assessments for Equality

1. What changes have occurred in your data from the baseline assessment to the summative? What do you think has impacted those changes?

2. How do items #2 and #5 compare this time? Is the pure equation or the word problem easier for students? What did you notice in the student work?

3. What items affirmed your instruction so far this year? What aspects of your instruction do you think influenced those items?

4. What still remains problematic for students? What are your plans for continuing to improve student work in this area?

Kaitlin Problems

At what grade would you expect most students
to be able to do each of the following problems?

A. Kaitlin had 13 cookies. She ate 6 of them. How many cookies does Kaitlin have left?

B. Kaitlin has 3 packages of gum. There are 6 pieces of gum in each package. How many pieces of gum does Kaitlin have altogether?

C. Kaitlin has 7 dollars. How many more dollars does she have to earn to have 11 dollars to buy a puppy?

D. Kaitlin had 15 guppies. She put 3 guppies in each jar. How many jars did Kaitlin put guppies in?

E. Kaitlin has 12 balloons. A friend of hers has 7 balloons. How many more balloons does Kaitlin have than her friend?

F. Mr. Gomez had 20 cupcakes. He put the cupcakes into 4 boxes so that there were the same amount of cupcakes in each box. How many cupcakes did Mr. Gomez put in each box?

G. Nineteen children were taking a minibus to the zoo. They will have to sit either 2 or 3 to a seat. The bus has seven seats. How many children will have to sit three to a seat, and how many can sit two to a seat?

H. Kaitlin had 3 packages of cupcakes. There were four cupcakes in each package. She ate 5 of the cupcakes. How many cupcakes were left?

I. Nineteen children are going to the circus. Five children can ride in each car. How many cars will be needed to get all nineteen children to the circus?

Excerpts from *Modeling Word Problems*

Minnesota STEM Teacher Center Frameworks http://www.scimathmn.org/stemtc/frameworks
→ Resources → Mathematics Best Practices → Modeling Word Problems

Overview

Using models is a critical step in helping students transition from concrete manipulative work with word problems to the abstract step of generating an equation to solve contextual problems. By learning to use simple models to represent key mathematical relationships in a word problem, students can more easily make sense of word problems, recognize both the number relationships in a given problem and connections among types of problems, and successfully solve problems with the assurance that their solutions are reasonable.

> *The failure to capture the mathematics being taught with a picture that helps students visualize what is going on is one of the most serious missed opportunities I observe.* Leinwand, 2009, p.19

Importance

Why is modeling word problems important?

Mr. Alexander and teachers from his team were talking during their Professional Learning Community (PLC) meeting about how students struggle with word problems. Everyone felt only a few of their students seem to be able to quickly generate the correct equation to solve the problem. Many students just seem to look for some numbers and do something with them, hoping they solve the problem. **read more**

What is modeling word problems?

Models at any level can vary from simple to complex, realistic to representational. Young students often solve beginning word problems, acting them out, and modeling them with the real objects of the problem situation, e.g. teddy bears or toy cars. Over time they expand to using representational drawings, initially drawing pictures that realistically portray the items in a problem, and progressing to multi-purpose representations such as circles or tally marks. After many concrete experiences with real-life word problems involving joining and separating, or multiplying and dividing objects, teachers can transition students to inverted-V and bar model drawings which are multi-purpose graphic organizers tied to particular types of word problems

Modeling Basic Number Relationships

Simple diagrams, sometimes known as fact triangles, math mountains, situation diagrams, or representational diagrams have appeared sporadically in some curriculum materials. But students' problem solving and relational thinking abilities would benefit by making more routine use of these diagrams and models. **read more**

Models and Problem Types for Computation

As children move to multi-digit work, teachers can transition students to bar model drawings, quick sketches that help students see the relationships among the important numbers in a word problem and identify what is known and unknown in a situation. **read more**

Planning and Instruction

How do I intentionally plan for and use modeling?

If modeling is not a way you learned to identify the important information and numerical relationships in word problems, you may want to review some of the resources on problem types (see Carpenter's book in References and Resources section below), or bar modeling (see books by Forsten, Walker, or Yeap in the References and Resources section below). You may also want to practice the different types of models. Decide which are most accessible for your students, and start with introducing one model at a time, helping students determine what is unknown in the problem, and where that unknown and the other numerical information should be placed in the bar model. A question mark, box, or a variable can be used for the unknown. As students become comfortable with that model, introduce, and compare and contrast a second model with the known model. **read more**

Summary

Several studies have shown that students who can visualize a word problem through modeling increase their problem solving ability and accuracy. This has been particularly documented in Singapore and other high performing countries where bar modeling is used extensively across grades. Students are more likely to solve problems correctly when they incorporate bar model drawings. On difficult problems, students who have been able to easily generate equations with simple problems often find that bar model drawings are especially helpful in increasing accuracy as problems increase in difficulty or involve new concepts (Yeap, 2010, pp. 87-89).

TALK: Reflection and Discussion

1. Are there particular types of word problems that your students solve more easily than others? What characterizes these problems?
2. Identify some basic facts with which your students struggle. How could you incorporate those facts into word problems, and how might the use of the inverted-V or bar model help?
3. How do bar model drawings help extract and represent the mathematical components and numerical relationships of a word problem?
4. With which type of word problems would you begin to show your students the use of bar model drawings?

DO: Action Plans

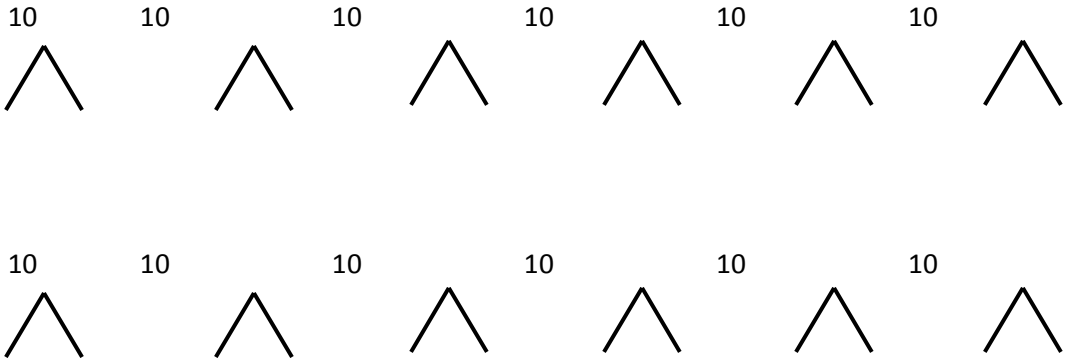
1. Select several story problems from your curriculum, MCA sample test items, or the Forsten, Walker, or Yeap resources on bar model drawing. Practice creating a bar model for several problems. Compare your models with others in your grade level, team, or PLC group. Practice until you feel comfortable with various model drawings.
2. Investigate the types of multiplication and division problems, and how bar models can be used with different types such as measurement and partitive division, arrays, equal groups, rates. The Carpenter resource may be helpful.
3. Select some problems from your curriculum that are of a similar type. Which bar model would be helpful in solving this type of problem? Practice using the model yourself with several problems of this type. How will you introduce the model to your students?
4. Identify some basic facts with which your students struggle. Craft some rich word problems utilizing these fact families. Introduce the inverted-V diagrams with the word problems to make sense of the information in the word problem, and discuss strategies for solving the problems.
5. Initiate a "Word-Problem-of-the-Day". Students might want to keep problem solving notebooks. Begin with problems of a particular type, and show students how to use a bar model to represent the information in a problem. Cluster several problems of a given type during the week. What improvements do you see in student selection of appropriate equations, accuracy of solutions, and ability to estimate or justify their answers as they increase the use of bar models to solve the word problems? A quick way to disseminate the "Word-Problem-of-the-Day" is to duplicate the problem on each label on a sheet of address labels. Students can just peel off the daily problem, add it to their problem solving notebook or a sheet of paper and solve away.
6. When your district is doing a curriculum materials review, advocate to include a criteria that requires the use of visual models in helping students make sense of mathematical problems.
7. Watch some of the videos of students using models on the Powerful Practices CD (see Carpenter and Romberg in References and Resources Section).

> *Word problems require that students have the skills to read, understand, strategize, compute, and check their work. That's a lot of skills! Following a consistent step-by-step approach-and providing explicit, guided instruction in the beginning - can help our students organize their thoughts and make the problem-solving task manageable.* Forsten, 2010, p.1

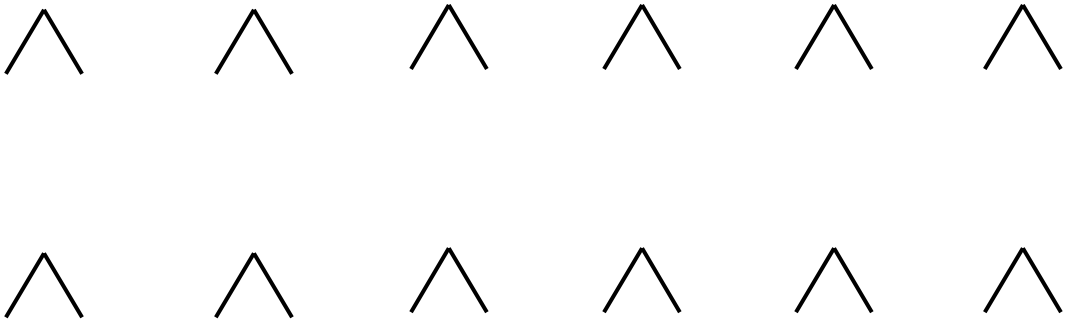
The Inverted "V" or "Mountain"

"Breaking Apart" Numbers or Expressions: A Part of Thinking Algebraically

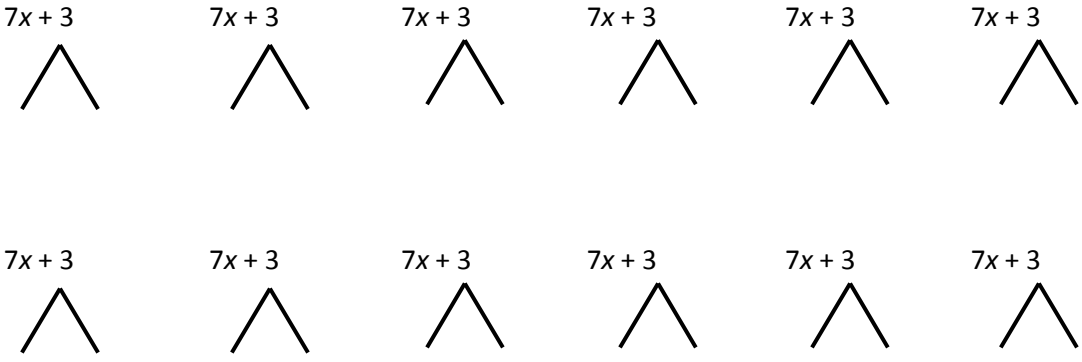
Make 10

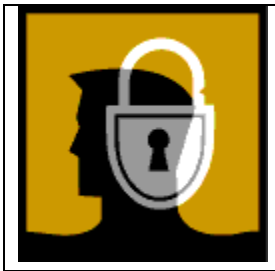


(design your own)



Algebraic Expressions





Unlocking Important Information in a Word Problem

	Read problem and highlight question.
	Write an answer sentence.
	Reread problem. Stop at key check points. Decide how to use the information – how it relates to the rest of the information.
	Create a visual model to show the relationships among the numerical pieces of information.
	Use the model to solve the problem and/or generate an equation to solve the problem.
	Record your answer with an appropriate label.
	SUPERCHECK – Does the answer make sense?

"Just Right" Word Problems: An easy way to differentiate.

Students select the numbers that are "just right" for them.

1. Sam has _____ boxes of baseball cards.
   He gives _____ boxes to his friend, Jesse.
   How many does he have left?

   (353, 122) (342, 175) (672, 39) or

   (6½, ½) (6½, ¾) (6½ , 2¾)

2. Bethann has a seashell collection.
   She has _____ white and _____ spotted shells.
   How many shells does she have altogether?

   (8, 34) (552, 329) (3,937, 3,487)

3. Tameka rode on _____ Valley Fair rides last weekend.
   Her friend, Jan rode on _____ rides.
   How many more rides did Tameka have than Jan?

   (27, 21) (56, 47) (83, 56)

Problem of the Week:
Making structures and relationships more transparent

Select one to three word problems. Each day change something in the original problem (numbers, names, contexts). Have students work the problems at a set time each day (e.g. coming into school, after recess or a specialist class).

Discuss similarities from one day to the next.

Textbook Search

Find some problems that exist in your curriculum materials. What number sets might work well to differentiate the problems for your students? You may want to use the numbers given in the problem as your middle set. Then **make the problem more accessible** by crafting an easier set of numbers for the first set and for the third set, crafting a set of numbers to **challenge students or extend their thinking**.

You might want to write the number sets on a post it and stick next to the problem in your teacher's manual to record good sets of numbers.

For example, consider this problem from curriculum materials: Alexis scored 12 points and Quincy scored 6 points. If their team scored 41 points, how many points did the rest of the team score? (EM 3, 7-4)	(12, 8, 30) (12, 6, 41) (18, 26, 93)

You may also want to craft problems that help students **develop relational thinking**. For example, consider the problem, *Zach had 70 pet mice. Zach gave 13 mice to his brother, Cole. How many mice does Zach have now?* The number sets (10, 70, 700) and (3, 13, 413) were crafted so the ones place is always 10 – 3. You could ask students, *You all used different number pairs today but why did everyone get a 7 in the one's place?*

Classification of Addition & Subtraction Word Problems

Although there are a number of ways that word problems can be distinguished from each other, one of the most useful ways of classifying them focuses on the types of action or relationships described in the problems. This classification corresponds to the way that children think about problems. Carpenter, et.al, 1999, p. 7

Basic Type		CGI Problem Type	Problem Types – students will solve problems differently depending on what is unknown and their own developmental level in mathematical understanding. (Numbers in parenthesis are for older learners.)		
Part-Part-Whole (Total)	**Start-Change-End** ***problems with action***	**Join**	***(Result Unknown)*** Connie had 5 (75, 758) marbles. Juan gave her 8 (89, 895) more marbles. How many marbles does Connie have altogether?	***(Change Unknown)*** Connie had 5 (75, 758) marbles. How many more marbles does she need to have 13 (164, 1653) altogether?	***(Start Unknown)*** Connie had some marbles. Juan gave her 8 (89, 895) more marbles. Now she has 13 (164, 1653) marbles. How many marbles did Connie have to start with?
		Separate	***(Result Unknown)*** Connie had 13 (164, 1653) marbles. She gave 5 (75, 758) to Juan. How many marbles does Connie have left?	***(Change Unknown)*** Connie had 13 (164, 1653) marbles. She gave some to Juan. Now she has 5 (75, 758) marbles left. How many marbles did Connie give to Juan?	***(Start Unknown)*** Connie had some marbles. She gave 5 (75, 758) to Juan. Now she has 8 (89, 895) left. How many marbles did Connie have to start with?
	inactive sets	**Part-Part Whole**	***(Whole Unknown)*** Connie has 5 (75, 758) red marbles and 8 (89, 895) blue marbles. How many marbles does she have?	***(Part Unknown)*** Connie has 13 (164, 1653) marbles. 5 (75, 758) are red and the rest are blue. How many blue marbles does Connie have?	
Comparison **Or Quantity-Quantity-Difference**		**Compare**	***(Difference Unknown)*** Connie has 13 (164, 1653) marbles. Juan has 5 (75, 758) marbles. How many more marbles does Connie have than Juan?	***(Compare Unknown) (Greater Amount Unknown)*** Juan has 5 (75, 758) marbles. Connie has 8 (89, 89) more marbles than Juan. How many marbles does Connie have?	***(Referent Unknown) (Lesser Amount Unknown)*** Connie has 13 (164, 1653) marbles. She has 5 (75, 758) more marbles than Juan. How many marbles does Juan have?

Basic problem types synthesized with Carpenter, Fennema, Franke, Levi, Empson. (1999). *Children's Mathematics: Cognitively Guided Instruction (CGI).* Portsmouth, NH: Heinemann Books. www.heinemann.com

✪✪✪✪ Part-Part-Whole (Total) Problems

Part-Part-Whole, Whole (Total) Unknown (or Start-Change-End, Result Unknown)

1. Zachary had _____ baseball cards.
   Zachary got _____ more baseball cards.
   How many cards does Zachary have now?
   (4, 3) (24, 21) (376, 248) (12½, 6¾ (*boxes of*))

Answer Sentence

Answer

_____ _____

(label)

Model **Workspace/Equation**

Part-Part-Whole, Part Unknown (or Separate Problem – Result Unknown)

2. Zach had _____ baseball cards.
   Zach gave _____ baseball cards to his brother, Cole.
   How many cards does Zach have now?
   (10, 3) (70, 13) (700, 413) (22¾ , 6½ (*boxes of*))

Answer Sentence

Answer

_____ _____

(label)

Model **Workspace/Equation**

Part-Part-Whole, Part Unknown (or Join Problem – Change Unknown)

3. Zach had _____ baseball cards.
Zach got some more cards at the store.
Now Zach has _____ cards.
How many cards did Zach get at the store?
(5, 12) (43, 73) (582, 731) (36¾ , 43½ (*boxes of)*)

Answer Sentence

Answer

_____ _____
(label)

Model **Workspace/Equation**

Part-Part-Whole, Whole (Total) Unknown (or Separate Problem – Start Unknown)

4. Zach had some baseball cards.
Zach gave _____ cards to his brother, Cole.
Now Zach has _____ baseball cards.
How many cards did Zach have at the beginning?
(4, 8) (25, 52) (155, 363) (24.75, 52.25 (*boxes of)*)

Answer Sentence

Answer

_____ _____
(label)

Model **Workspace/Equation**

Percent, Multi-Step, Part-Part-Whole, Part Unknown

5. Riley had 200 stamps.
   - 35% are from Europe
   - 10% are from Asia
   - 20% are from Australia

   The rest of the stamps are from North America. How many of Riley's stamps are from North America?

   A. 35 **B**. 65 *__C__. 70 **D.** 130

   (MCA III Item Sampler, Gr. 6, 2012)

Answer Sentence

Answer

_____ _____

(label)

Model **Workspace/Equation**

Percent, Multi-Step, Part-Part-Whole, Part Unknown

6. A company is printing 250 calendars. In 1 hour, 75 calendars are printed. What percent of the calendars are printed in 1 hour?

   A. 3% **B**. 3.3% *__C__. 30% **D.** 33%

Answer Sentence

Answer

_____ _____

(label)

Model **Workspace/Equation**

PART-PART-WHOLE PROBLEMS ✪✪✪✪ *TRY IT!*

You can change the numbers to "just right" numbers if you like.

7. Part-Part-Whole, Whole Unknown There are 23,650 people in a stadium. The stadium can hold 1,000 more people. How many people can the stadium hold? (MCA III Item Sampler, Gr. 3, 2012)	Model and Workspace
Answer Sentence	

8. Part-Part-Whole, Part Unknown Pat and Tracy collected a total of 576 aluminum cans. Pat collected 398 cans. How many did Tracy collect? (MCA II Item Sampler)	Model and Workspace
Answer Sentence	

9. Part-Part-Part-Whole, Part Unknown Dana played a total of 28 soccer games during the months of June, July and August. She played 8 games in June and 11 games in July. How many games did Dana play in August? **(**MCA II Item Sampler)	Model and Workspace
Answer Sentence	

10. Part-Part-Whole, Part Unknown, Unknown as a Variable (or Separate, Change Unknown) Tom had 18 pencils. He gave *n* pencils away and had 2 pencils left over. How many pencils did Tom give away? (adapted from MCA III Item Sampler, Gr. 3, 2012)	Model and Workspace
Answer Sentence	

11. Multi-step: Part-Part-Whole, Part Unknown Brad bought 5 sports drinks for $1.25 each. He gave the cashier a $20.00 bill. How much should Brad receive in change? (MCA II Item Sampler)	Model and Workspace
Answer Sentence	

12. Multi-step: Part-Part-Whole – Parts Unknown Jeff had 1,350 glass beads and 695 clay beads. He sold 138 glass beads and 47 clay beads. How many beads did Jeff have left? (MCA III Item Sampler, Gr. 3, 2012)	Model and Workspace
Answer Sentence	

13. Multi-step: Part-Part-Whole, Part Unknown, One quantity related to the other Andrew and Thomas collected a total of 190 bugs for a science project. Andrew collected 10 more bugs than Thomas. How many bugs did Andrew collect? (MCA II Item Sampler)	Model and Workspace
Answer Sentence	

14. Multi-step: Part-Part-Part-Part-Whole (or Part w/multiplication-Part-Whole), Whole Unknown A camping group bought 3 sleeping bags that cost $42 each and a tent that cost $160. What was the total cost of the sleeping bags and tent? (adapted from MCA III Item Sampler, Gr. 4, 2012)	Model and Workspace
Answer Sentence	

15. Part-Part-Whole, Whole Unknown A bookcase has 4 shelves. The bottom shelf has 10 books. Each of the other shelves has 5 more books than the shelf below it. How many books are in the bookcase? (MCA III Item Sampler, Gr. 5, 2012)	Model and Workspace
Answer Sentence	

16. Part-Part- Whole, writing an equation involving variables for a repeated addition or multiplication situation Leon plants 3 rows of tomatoes with *n* plants in each row. He also plants 1 row of beans with 5 plants in the row. Which equation can be used to find *t*, the total number of plants Leon planted? **A.** $t = n + 8$ **C.** $t = 3n + 5$ **B.** $t = 3n + 1$ **D.** $t = 5n + 3$ (MCA III Item Sampler, Gr. 8, 2012)	Model and Workspace
Answer Sentence	

17. Part-Part-Whole, writing an inequality involving variables Ann sells bracelets for $4 each and necklaces for $8 each. Which inequality shows *x, the* number of bracelets, and *y,* the number of necklaces Ann must sell to make at least $100? **A.** $4x + 8y \leq 100$ **C.** $8x + 4y \leq 100$ **B.** $4x + 8y \geq 100$ **D.** $8x + 4y \geq 100$ (MCA III Item Sampler, Gr. 8, 2012)	Model and Workspace
Answer Sentence	

18. Fractions, Part-Whole, Part Unknown Jason has 8 cupcakes.  He eats 1/8 of the cupcakes and gives 2/8 of the cupcakes to his friends. What fraction of the cupcakes are left? (MCA III Item Sampler, Gr. 4, 2012)	Model and Workspace
Answer Sentence	

19. Fractions, Part-Whole, Part Unknown Joe and Mari were painting 24 rows of the bleachers at their school. They painted ¼ of the bleachers each day for 3 days. How many rows were painted in the 3 days?	Model and Workspace
Answer Sentence	

20. Fractions, Multiple Parts-Whole, Whole Unknown ½ of a group are children. 1/3 of the adults are men. There are 36 women. How many people are in the group? (Yeap Ban Har, *Bar Modeling: A Problem-solving Tool.* Marshall Cavendish Education, 2010, p.134)	Model and Workspace
Answer Sentence	

21. Fractions, Part Whole, Part Unknown A grocer has 42 apples. 2/7 of them are red, and the rest are green. How many of them are green? (John Hoven and Barry Garelick, "Singapore Math: Simple or Complex?" *Educational Leadership*, November 2007.)	Model and Workspace
Answer Sentence	

22. Fractions, Part Whole, Parts Unknown Bob, Liz, and Eli drove from Chicago to Denver. They drove 1,050 miles altogether. Bob drove 1/10 of the distance, Liz drove 4/10, and Eli drove ½ of the distance. How many miles did each person drive? (EM, Gr. 5, Lesson 5.1, Math Journal p122, #2)	Model and Workspace
Answer Sentence	

23. Fractions & Percent, Part-Whole, Part Unknown Lauren spent 20 percent of her money on a dress. She spent 2/5 of the remainder on a book. She had $72 left. How much money did she have at first? (John Hoven and Barry Garelick, "Singapore Math: Simple or Complex?" *Educational Leadership*, November 2007.)	Model and Workspace
Answer Sentence	

24. Fractions, Multiple Parts-Whole, Whole Unknown 5/8 of a group of boys chose the lion as their favorite animal. 2/3 of the rest chose the monkey. The remaining 18 boys chose the giraffe. How many boys were there in the group? (Yeap Ban Har, *Bar Modeling: A Problem-solving Tool.* Marshall Cavendish Education, 2010, p.138)	Model and Workspace
Answer Sentence	

25. Decimals, Part-Whole, Part Unknown When a monarch butterfly is born it is very tiny caterpillar 0.35 cm long. It grows during its life to become 5.15 cm long. How much length did it gain after it was born? (*Math Expressions, Gr. 5.* Adapted from Class Activity 3.10, #8)	Model and Workspace
Answer Sentence	

26. Decimals, Part-Whole, Part Unknown Ahmad had a piece of rope that was 7.14 meters long. He cut off 0.095 meters to practice making knots. What was the length of the rope after the cut? (*Math Expressions,* Gr. 5, Homework and Remembering 3.9, #7)	Model and Workspace
Answer Sentence	

	***Skip over to pp. 44-45 to generate problems for your students, try some addition/subtraction SPILL THE BEANS problems with bar models or read the "Singapore Math: Simple or Complex" article.***

✪✪✪✪ Comparison Problems

Consider a line of equality with comparison problems*.*

Comparison, Difference Unknown

27. According to the 2010 Census, the population of Duluth was 86,265 and Brainerd had a population of 13,590. How many fewer people lived in Brainerd in 2010?

Answer Sentence

Answer

_____ _____

(label)

Model **Workspace/Equation**

Comparison, Lesser Quantity Unknown

28. Pam is 10 years younger than Margaret. Margaret is 32. How old is Pam? (MCA II Item Sampler)

Answer Sentence

Answer

_____ _____

(label)

Model **Workspace/Equation**

Multi-Step: Triple Comparison, Greater and Lesser Quantities Unknown + Part-Part-Part-Whole, Whole Unknown

29. Mai played 12 more video games than Becky. Tameka played 4 less games than Mai. How many games were played by the 3 girls if Becky played 24 video games?

Answer Sentence

Answer

_____ _____

(label)

Model **Workspace/Equation**

Comparison, Difference Unknown – Alternate Wording

30. Zach has 263 Pokemon cards. Cole has 139 Pokemon cards. How many more cards would Cole have to get to have the same number of cards as Zach?

Answer Sentence

Answer

_____ _____

(label)

Model **Workspace/Equation**

Comparison Problems ✪✪✪✪ *TRY IT!*

You can change the numbers to "just right" numbers if you like.

31. Comparison – Lesser Quantity Unknown Jack and Leila have each been saving money. Jack has $136.83. Leila has $46 less money saved. How much money has Leila saved? (Math Expressions, adapted from Gr. 5 Class Activity 3.9, #6)	Model and Workspace
Answer Sentence	

32. Comparison, Larger Quantity Unknown There are 2,387 people seeing a movie. There are 5,896 more people seeing a play. How many people are seeing a play? (Math Expressions, Gr. 5 Assessment Unit3, Quick Quiz 5)	Model and Workspace
Answer Sentence	

33. Comparison, Difference Unknown, Decimals One year the Sahara Desert received 0.791 inches of rain. That same year the rain forest in Brazil received 324 inches of rain. How much less rain fell in the desert than in the rain forest that year? (Math Expressions, Gr. 5, Class Activity 3.9, adapted from #1)	Model and Workspace
Answer Sentence	

34. Comparison, Lesser Quantity Unknown, Decimals Johan's race time was 45.03 seconds. Kyle's race time was 0.1 second less than Johan's time. What was Kyle's race time? (MCA III Item Sampler, Gr. 5, 2012)	Model and Workspace
Answer Sentence	

35. Comparison, Difference Unknown, Fractions Jill is $48\frac{5}{8}$ inches tall. Lei is 47.5 inches tall. What is the difference in their heights? (MCA III Item Sampler, Gr. 5, 2012)	Model and Workspace
Answer Sentence	

36. Comparison, Multi-Step, Lesser Quantity Unknown, writing equations for a given multi-step situation Lisa has 6 more green marbles than blue marbles. She has a total of 40 green and blue marbles. Write a system of equations that represents this situation if *x* is the number of green marbles and *y* is the number of blue marbles. How many blue marbles does Lisa have? (adapted from MCA III Item Sampler, Gr. 8, 2012)	Model and Workspace
Answer Sentence	

	*Skip over to pp. 43-44 to generate problems for your students, go back to any Part-Whole problems, try some +/– SPILL THE BEANS problems with bar models or read "Singapore Math: Simple or Complex" article.*

Multiplication and Division Problem Types

Equal Grouping – Asymmetrical Structure

• Multiplication

There are four basketball teams at the tournament and each team has five players. How many players are at the tournament?

• Measurement or Quotitive Division

I have 24 apples. How many paper bags will I fill if I put 3 apples into each bag?

• Partitive Division

Twenty-four apples need to be placed into eight paper bags. How many apples will you put in each bag if you want the same number in each bag?

Rate Problems – Asymmetrical Structure

A baby elephant gains 4 pounds each day. How many pounds will the baby elephant gain in 8 days?

A baby elephant gains 4 pounds each day. How many days will it take the baby elephant to gain 32 pounds?

A baby elephant gained 32 pounds in 8 days. If she gained the same amount of weight each day, how much did she gain in one day?

Price Problems – Asymmetrical Structure

How much would five pieces of bubble gum cost if each piece costs 4 cents?

Bubble gum costs 4 ¢ for each piece. How many pieces of bubble gum can you buy with 20¢?

If you can buy 5 pieces of bubble gum with 20 cents, how much does each piece cost?

Ratio Problems

- Part to Part

  The ratio of boys to girls in the school band is 3:2. If there are 27 boys, how many girls are in the band?

- Part to Whole

  There are 45 students in the band. The girls are 2/5 of the band. How many of the band members are boys?

Multiplicative Comparison Problems – Asymmetrical Structure

Catherine read 12 books. Elizabeth read 4 times as many. How many books did Elizabeth read?

Elizabeth read 48 books during summer vacation. Catherine read 12 books during summer vacation. How many times greater is the number of books Elizabeth read compared with the number of books Catherine read?

Elizabeth read 48 books during the summer vacation. This is 4 times as many as Catherine. How many books did Catherine read during summer vacation?

Area and Array Problems – Symmetrical Structure

- *Area*

  A baker has a pan of fudge that measures 8 inches on one side and 9 inches on the other side. If the fudge is cut into square pieces 1 inch on a side, how many pieces of fudge does the pan hold?

  A farmer plants a rectangular vegetable garden that measures 6 meters along one side and 8 meters along an adjacent side. How many square meters of garden did the farmer plant?

  A farmer plans to plant a rectangular vegetable garden. She has enough room to make the garden 6 meters along one side. How long does she have to have to make the adjacent side in order to have 48 square meters of garden?

- *Array*

  For the second-grade play, the chairs have been put into 4 rows with 4 chairs in each row. How many chairs have been put out for the play?

Combination/Cross Product Problems

Pete's Deli stocks four types of cold cuts and two types of cheese. How many different sandwiches consisting of one type one meat and one type of cheese are possible?

References:

For more complete descriptions of the different problem types and how children solve these kinds of problems, see:

Children's Mathematics, Cognitively Guided Instruction. *T. Carpenter, E, Fennema, M.L. Franke, L. Levi, S. B. Empson. 1999. Heinemann Press: Portsmouth, NH. ISBN 0-325-00137-5*

Math Matters Grades K-6; Understanding the Math You Teach. *S. H. Chapin, A. Johnson. 2000. Math Solutions Publications: Sausalito, CA. ISBN 0-941355-26-8*

Region 11 Algebra Academy, http://www.region11mathandscience.org/

✪✪✪✪ Multiplication & Division Problems

Multiplication – Equal Groups or Array, Product Unknown

1. The gym was set up for a concert. There were _____ chairs in each row for the players. There were _____ identical rows of chairs. How many chairs were set up for the band?

(9, 4) (9, 8) (18, 4)

Answer Sentence

Answer

_____ _____

(label)

Model **Workspace/Equation**

Multiplication – Product Unknown

2. A truck has 50 boxes of jump ropes. Each box contains 100 jump ropes. How many jump ropes are on the truck?

(MCA III Item Sampler, Gr. 4, 2012)

(18, 6) (108, 6) (108, 36)

Answer Sentence

Answer

_____ _____

(label)

Model **Workspace/Equation**

Multiplication/Division - Factor Unknown – Partitive Division

3. Malik has 64 marbles. He puts an equal number of marbles into each of 4 jars. How many marbles are in each jar? (MCA III Item Sampler, Gr. 3, 2012)

Answer Sentence

Answer

_____ _____

(label)

Model **Workspace/Equation**

Multiplication/Division - Factor Unknown – Measurement Division

4. There are 35 students going on a class trip. The students ride in vans. There are 7 students riding in each van. How many vans are needed to take all the students? What if there were 38 students?
(MCA III Item Sampler, Gr. 4, 2012)

Answer Sentence

Answer

_____ _____

(label)

Model **Workspace/Equation**

Multiplication/Division – Factor Unknown – Measurement Division

5. At the factory that makes and boxes colored markers, there is a bin that holds ___ markers. If the sorting machine puts ___ markers into each box, how many boxes can the machine fill?

Factory	A	B	C
	(561, 8)	(561,16)	(1683, 24)

Answer Sentence

Answer

_____ _____

(label)

Model **Workspace/Equation**

Multi-Step: Part-Part-Whole, Part Unknown (one part is given, the unknown part is multiplication with a factor unknown)

6. Robert has 54 pencils. He has 1 box of pencils and 3 packages of pencils. The box has 24 pencils. How many pencils are in each package, if each package has the same number of pencils?
(adapted from MCA III Item Sampler, Gr. 4, 2012)

Answer Sentence

Answer

_____ _____

(label)

Model **Workspace/Equation**

MULTIPLICATION & DIVISION ✪✪✪✪ TRY IT!

You can change problems to "just right" problems if you wish.

7. Multiplication, Equal Groups, Product Unknown Jason bought _____ video games at the store. Each game cost _____. How much did Jason spend before the tax was added on? (6, $12) (6, $29.99) (12, $29.99)	Model and Workspace
Answer Sentence	

8. Multiplication, Equal Groups, Product Unknown, Days in a particular month Sam runs 32 km a day during April to get ready for a race. If Sam runs every day of the month, how many total kilometers did he run in April? (SciMathMN Frameworks)	Model and Workspace
Answer Sentence	

9. Multi-Step: Part-Part-Whole, Whole Unknown (one part is multiplication, product unknown) A camping group bought 5 sleeping bags that cost $42 each and a tent that cost $160. What was the total cost of the sleeping bags and tent? (adapted from MCA III Item Sampler, Gr. 4, 2012)	Model and Workspace
Answer Sentence	

10. Multiplication/Division, Factor Unknown, Measurement Division, Remainders Jan has 500 pieces of paper. She prints as many copies as possible of a 16-page report. How many pieces of paper are left? (MCA III Item Sampler, Gr. 5, 2012)	Model and Workspace
Answer Sentence	

11. Multiplication, Product Unknown (non-whole numbers) Stephanie is making butterfly wings for the school play. She needs to make 10 pair of wings. Each pair needs 1 ½ yards of fabric. How many yards of fabric should she buy to make 10 pairs of wings?	Model and Workspace
Answer Sentence	

12. Multiplication/Division – Factor Unknown – Measurement Division, use of variable Alice bought *n* books and spent $18. Each book cost $2. How many books did Alice buy? (adapted from MCA III Item Sampler, Gr. 3, 2012)	Model and Workspace
Answer Sentence	

	*Skip over to pp. 43-44 to generate problems for your students, go back to any previous problems, try some +/– SPILL THE BEANS problems with bar models, or read "Singapore Math: Simple or Complex" article.*

✪✪✪✪ Multiplicative Comparisons - *Think Equal Units*

Multiplicative Comparison – Greater Quantity Unknown

13. Julio had 25 baseball cards. His friend, Mario, had 5 times as many cards as Julio. How many cards did Mario have?
Answer Sentence
Answer _____ _____ **(label)**
Model **Workspace/Equation**

Multiplicative Comparison – Lesser Quantity Unknown

14. Rafael has 75 Pokemon cards. His friend, Jamie, has one-third as many cards as Rafael. How many cards does Jamie have?
Answer Sentence
Answer _____ _____ **(label)**
Model **Workspace/Equation**

Multiplicative Comparison – Comparison Unknown

15. Juanita picked 21 buckets of strawberries. Pam picked 7 buckets of strawberries. How many times greater is the number of buckets Juanita picked compared to the number of buckets Pam picked?

Answer Sentence

Answer

_____ _____

(label)

Model **Workspace/Equation**

Multi-Step: Multiplicative Comparison & part-Part-Whole

16. Arielle has one-third the friendship bracelets that Pai has. Together they have a total of 48 bracelets. How many bracelets does Arielle have?

Answer Sentence

Answer

_____ _____

(label)

Model **Workspace/Equation**

MULTIPLICATIVE COMPARISON ✪✪✪✪ TRY IT!

17. Multiplicative Comparison – Comparison Unknown Brian ran 12 laps around the track. Marcus ran 48 laps. How many times less did Brian run compared to Marcus?	Model and Workspace
Answer Sentence	

18. Multiplicative Comparison – Greater Quantity Unknown There are twice as many girls as boys in choir. If there are 36 students in the choir, how many are girls?	Model and Workspace
Answer Sentence	

19. Multi-Step: Additive & Multiplicative Comparison – Greater Quantity Unknown Athena is 4 years older than Brittany. Carlota is twice as old as Brittany. If Athena is 13 years old, how old is Carlota? How old are the three girls altogether?	Model and Workspace
Answer Sentence	

20. Multiplicative Comparison – Greater Quantity Unknown The sum of three numbers (A, B, C) is 120. A is the largest number and it is three times as large as C. B is twice as large as C. How much is A?	Model and Workspace
Answer Sentence	

	*Skip over to pp. 43-44 to generate problems for your students, go back to any previous problems, try some +/– SPILL THE BEANS problems with bar models or read "Singapore Math: Simple or Complex" article.*

✪✪✪✪ Ratio and Rate Problems

Ratio: Part to Part

21. The ratio of boys to girls in the Chess Club is 2:3. If there are 65 students in the Club, how many are girls?
Answer Sentence
Answer _____ _____ **(label)**
Model **Workspace/Equation**

Ratio: Unit Rate

22. Wes bought two baseballs for $5. How much will seven baseballs cost? (Region 11 Algebra Academy, 2010)
Answer Sentence
Answer _____ _____ **(label)**
Model **Workspace/Equation**

Ratio: Scale Factor

23. The action figure of a famous wrestler has a scale factor of 1/12. If the action figure is 7 inches tall, how tall is the actual wrestler?
(Region 11 Algebra Academy, 2010)

Answer Sentence

Answer

_____ _____

(label)

Model **Workspace/Equation**

Ratio: Part to Part and Part to Whole

24. Jane looked at two kinds of trail mix. Brand A has 2 parts peanuts to 3 parts raisins. Brand B has 4 parts peanuts to 4 parts raisins. Jane wants to buy 1 pound of trail mix. Jane loves peanuts. Which brand has the higher concentration of peanuts?
(adapted from MN Academic Standards, Benchmark 6.1.2.2,, e.g.)

Answer Sentence

Answer

_____ _____

(label)

Model **Workspace/Equation**

Ratio: Part to Part to Part

25. Gail made muffins for a party by mixing cups of muffin mix, blueberries and water in a 6:2:1 ratio. Gail started with 18 cups of muffin mix. How many cups of blueberries did Gail use?

Answer Sentence

Answer

_____ _____

(label)

Model **Workspace/Equation**

Ratio: Part to Part and Part to Whole

26. A paint color is made using 4 drops of red and 5 drops of blue for each 5 gallons of paint. How many gallons of paint are being colored when 45 drops of color are used?
(MCA III, Gr. 6 Item Sampler)
*25 gallons

Answer Sentence

Answer

_____ _____

(label)

Model **Workspace/Equation**

RATIO & RATES ✪✪✪✪ TRY IT!

27. Ratio: Part to Part and Part to Whole The ratio of Maria's DVDs to Jana's DVDs is 3:5. If they have 32 DVDs altogether, how many DVDs does Jana have? (MCA III Item Sampler, Gr. 4, 2012)	Model and Workspace
Answer Sentence	

28. x/÷ Unit Rate, decimals A bottle of soap costs $3.45 for 64 ounces. What is the cost per ounce? (MCA III Item Sampler, Gr. 6, 2012) ***A.**$0.05 **B**.$0.19 **C**.$0.22 **D.**$0.64	Model and Workspace
Answer Sentence	

29. x/÷ Unit Rate, Decimals, Converting ounces to pounds Joleen bought 12 apples. Each apple weighed 1.8 ounces. How many pounds of apples did Joleen buy? (MCA III Item Sampler, Gr. 6, 2012) *** A.** 1.35 pounds **C**. 21.6 pounds **B**. 2.4 pounds **D.** 28.8 pounds	Model and Workspace
Answer Sentence	

30. x/÷ Rate, including Additive Comparison Kelly makes 12 candles in 3 hours. Lee makes 6 candles in 1 hour. What is the difference in the number of candles they each make in 8 hours? (adapted from MCA III Item Sampler, Gr. 6, 2012) *16 candles	Model and Workspace
Answer Sentence	

31. x/÷ Rate Jeremy can plant 10 trees in 4 hours. How many trees can he plant in 10 hours? (adapted from MCA III Item Sampler, Gr. 7, 2012) *25 trees	Model and Workspace
Answer Sentence	

32. x/÷ Rate/Scaling, Multi-Step, Factor Unknown A map uses the scale 1.5 cm = 25 miles. Two cities are 190 miles apart. How far apart are the cities on the map? (MCA III Item Sampler, Gr. 7, 2012) **A.** 0.21 cm **C**. 2,917 cm * **B**. 11.4 cm **D.** 6,563 cm	Model and Workspace
Answer Sentence	

33. x/÷ Rate, Using Equations, Multi-Step, Factor Unknown

The equation $y = 12x + 60$ can be used to estimate y, the height of a tree in centimeters x months after it is planted. When a tree is 150 cm tall, how long ago was the tree planted?

(MCA III Item Sampler, Gr. 7, 2012)

*** A.** 7.5 months **C**. 17.5 months

B. 10.8 months **D.** 78.0 months

Answer Sentence

Model and Workspace

34. x/÷ Multi-Step, 2-Part Rate, incorporating Comparison and Reading Tables

At a movie store, Erin pays a monthly fee and is charged for each movie she rents. The table shows the monthly cost when Erin rents different numbers of movies.

Monthly Cost	
Number of Movies	**Total Cost (dollars)**
6	33
8	39
10	45

How much is the monthly fee that Erin pays?

(MCA III Item Sampler, Gr. 5, 2012)

A. $3 **B**. $6 ***C**. $15 **D.** $18

Answer Sentence

Model and Workspace

35. x/÷ Use of Equation/Formula, 2-Part Rate, including Part-Whole and Factor/Divisor Unknown, Decimals A phone company uses the equation $y = 0.15x + 10$ to find y, the monthly charge for a customer sending x text messages. How many text messages are sent if the monthly charge is $77.50? (adapted from MCA III Item Sampler, Gr. 6, 2012) *450 messages	Model and Workspace
Answer Sentence	

36. x/÷ Rate, Multi-Step, incorporating Part-Whole, Part Unknown, Fractions, Decimals Nora is running a race that is 26.2 miles. She is running at a speed of 8 miles per hour. She has completed ¾ of the race. How much longer will it take Nora to finish the race? (MCA III Item Sampler, Gr. 7, 2012) * **A.** 0.82 hour **C**. 3.28 hours **B**. 2.46 hours **D.** 6.5 hours	Model and Workspace
Answer Sentence	

37. x/÷ Ratio, Decimals, Factor Unknown The equation $3c = 4s$ gives the relationship between c, the weight of clay, and s, the weight of sand in a mixture. There are 6.25 pounds of clay in the mixture. What is the weight of the sand? (MCA III Item Sampler, Gr. 7, 2012) * **A.** 4.69 pounds **C**.18.75 pounds **B**. 8.88 pounds **D.** 75.00 pounds	Model and Workspace
Answer Sentence	

38. x/÷ Rate/Scaling, Multi-Step, Factor Unknown

The table shows the cost of different numbers of boxes of cookies.

Selling Cookies	
Boxes of Cookies	**Cost (dollars)**
5	11.29
7	15.75
11	24.75

What is the cost to buy 15 boxes of cookies?

(MCA III Item Sampler, Gr. 7, 2012)

*** A. $33.75** **C**. $40.50

B. $36.00 **D.** $51.75

Answer Sentence

Model and Workspace

39. x/÷ Multi-Step, Average Rate of Change, incorporating Part-Whole, Difference Unknown and Equal Groups, Factor Unknown, Decimals

In 1977, there were 12,168,450 U.S. households with cable television. In 1997, there were 65,929,420 U. S. households with cable television. Over that time period, what was the average rate of change per year in households with cable television?

(MCA III Item Sampler, Gr.11, 2012)

*** A.** 2,688,048.5 households/year

B. 5,376,097.0 households/year

C. 53,760,970.0 households/year

D. 65,320,997.5 households/year

Answer Sentence

Model and Workspace

40. x/÷ Multi-Step, Average Rate of Change, incorporating Part-Whole, Difference Unknown and Equal Groups, Factor Unknown, Rounding Harrison High School has 768 students. In 6 years, it is projected to have 1,157 students. What is the projected average rate of change per year in students over this time period? Round your answer to the nearest student. (MCA III Item Sampler, Gr.11, 2012) *65 students	Model and Workspace
Answer Sentence	

41. x/÷ Ratio, Decimals, Factor Unknown Rosa wants to use $20 to buy games. The inequality $2.50k + 5.5 \leq 20.00$ represents the number of games, k, she can buy with her money. What is the greatest number of games Rosa can buy? (MCA III Item Sampler, Gr. 11, 2012) * **A.** 5 games **C**. 8 games **B**. 6 games **D.** 10 games	Model and Workspace
Answer Sentence	

✪✪✪✪ *ON YOUR OWN:*

Think of problem situations and number magnitudes appropriate for your students. How do bar models work with your problems?

Problem	**Model and Workspace**
Answer Sentence	

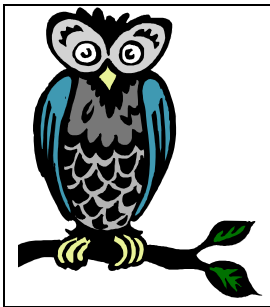
Problem	**Model and Workspace**
Answer Sentence	

Problem	**Model and Workspace**
Answer Sentence	

Problem	Model and Workspace
Answer Sentence	

Problem	Model and Workspace
Answer Sentence	

Problem	Model and Workspace
Answer Sentence	



Insights

1. How do bar models help establish the relationships among the numbers in a word problem?

2. How is this similar to or different from how you learned to do word problems or the strategies you have used to teach students to solve word problems?

3. What benefits do you see to using bar models?

4. How will you begin to use bar models with your students? (see also "DO: Action Plans" on p. 6 of this handout from SciMathMN Frameworks)

References

Bruner, J. S. (1961). The act of discovery. *Harvard Educational Review, 31,* pp. 21-32, in Yeap, Ban Har. (2010). *Bar modeling: A problem solving tool.* Singapore: Marshall Cavendish Education.

Carpenter, T. P., Fennema, E., Franke, M., Levi, L. & Empson, S. B. (1999). *Children's mathematics: Cognitively guided instruction.* Portsmouth, NH: Heinemann. (Book and CD)

Carpenter, T. P. & Romberg. T. A. (2004). Modeling, generalization, and justification in mathematics cases, in *Powerful practices in mathematics & science.* Madison, WI: National Center for Improving Student Learning and Achievement in Mathematics and Science. http://www.wcer.wisc.edu/ncisla (Booklet and CD)

Dienes, Z. (undated). *Zoltan Dienes' six-stage theory of learning mathematics.* Retrieved from http://www.zoltandienes.com

Everyday Mathematics. The Wright Group - McGraw Hill. https://www.mheonline.com/discipline/narrow/1/16/231/math

Forsten, C. (2009). *Step-by-step model drawing: Solving math problems the Singapore way.* Peterborough, NH: Crystal Spring Books. http://www.crystalspringsbooks.com

Hoven, J. & Garelick, B. (2007). Singapore math: Simple or complex? *Educational Leadership, 65 (3),* 28-31.

Leinwand, S. (2009). *Accessible mathematics: 10 instructional shifts that raise student achievement.* Portsmouth, NH: Heinemann.

Marzano, R. J., Norford, J. S., Paynter, D. E., Pickering, D. J., & Gaddy, B. B. (2001). *A handbook for classroom instruction that works.* Alexandria, VA: Association for Supervision and Curriculum Development.

Math Expressions. Houghton Mifflin Harcourt. http://www.eduplace.com/math/mthexp/

Math in Focus. Houghton Mifflin Harcourt. http://www.hmheducation.com/singaporemath/

Region 11 Algebra Connected to Number. (2010), Region 11 Math & Science Teacher Partnership. http://www.region11mathandscience.org/

Singapore Ministry of Education. (1997). Retrieved http://moe.gov.sg

Skemp, R. R. (January, 1993). "Theoretical foundations of problem solving: A position paper." University of Warwick. Retrieved from http://www.grahamtall.co.uk/skemp/sail/theops.html

Spill the Beans Activity Cards (Junior and Challenge versions). Crystal Springs Books. http://www.crystalspringsbooks.com or http://catherinekuhns.com/booksandgames.html

Walker, L. (2010). *Model drawing for challenging word problems: Finding solutions the Singapore way.* Peterborough, NH: Crystal Spring Books. http://www.crystalspringsbooks.com

Yeap, B. H. (2010). *Bar modeling: A problem solving tool.* Singapore: Marshall Cavendish Education. http://www.singaporemath.com

Yeap, B.H. and L. Walker (2012). *Every Child Can Do Math: Deceptively simple activities to develop mathematical thinking.* Peterborough, NH Crystal Spring Books. http://www.crystalspringsbooks.com