

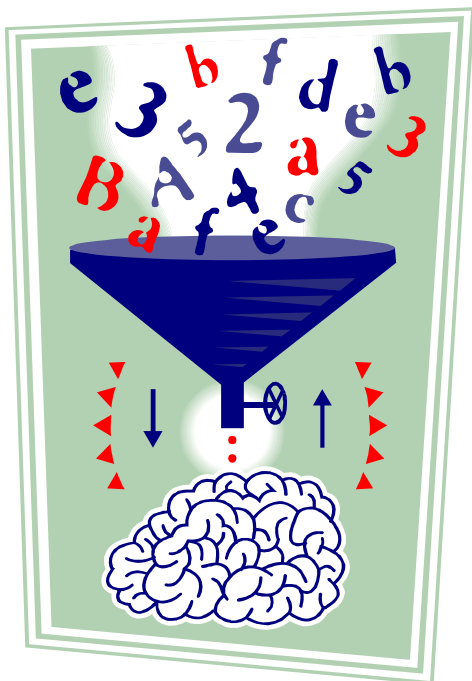
# 2012-2013 Algebra Academy

Exploring Student's Mathematical Thinking

Probing the Math Needed for Algebra

*for Special Education Staff of*

*District 287 & Member Districts*



Wednesday, Sept. 19 (Room 321)

**EQUALITY**

Wednesday, Nov. 7 (Room 321)

**MODELING WORD PROBLEMS**

Wednesday, Dec. 12 (Room 321)

**RELATIONAL THINKING**

Wednesday, Jan. 16 (Room 321)

**OPERATIONS & BASIC FACTS**

Wednesday, Feb. 27 (Room 321)

**FRACTIONS AND DECIMALS**

Wednesday, May 1 (Room 321)

**EFFORT, PROBLEM SOLVING &  
REASONING**

Facilitators: Nancy Nutting

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## Make a Note . . .

*287 Mathematics Website - Algebra Academy*

*Check for session handouts, assessments, templates & other resources*

<http://courses.district287.org/mod/page/view.php?id=10229>

*Krypto* <http://mphgames.com>


*Illuminations (NCTM)*

*Primary Krypto:* <http://illuminations.nctm.org/ActivityDetail.aspx?ID=173>

*Pan Balances:* <http://illuminations.nctm.org/ActivityDetail.aspx?ID=26>

(search also for Shape and Expressions Pan Balances)

*Sources for Pan and Number Balances:* about \$13-\$23 each,  
Nasco.com, amazon.com, WonderBrains.com, eaieducation.com (best price)

AGENDA	INSTRUCTIONAL STRATEGIES
<p>Overview of Academy Talking Trios</p> <p>What is Mathematical Power? Equal 9 &amp; Krypto Students and the Equal Sign Benchmarks for Equal Sign Video Clip - Kevin Balancing Expressions Video Clip - 4<sup>th</sup> Grade Class Equation Chains</p> <p><i>Lunch</i></p> <p>The Clothespin Card True/False Equations Open Number Sentences (Make it True) Equal or Not Equal</p> <p>Assessments PLC Structure</p> <p>Standards by Progression SciMath<sup>MN</sup> Frameworks for Math Feedback</p>	<ul style="list-style-type: none"> <li>• Krypto/Equal # posters</li> <li>• Increasing the variety of equation formats</li> <li>• Using "is the same as" language along with "equals"</li> <li>• Using balances</li> <li>• Using equation chains</li> <li>• Using clothespin cards</li> <li>• Using true/false &amp; open number sentences (make it true)</li> <li>• Using = and <math>\neq</math></li> </ul> <div style="text-align: center;">  </div>

Program Facilitator for Professional Learning  
Jennifer Nelson [jlnelson@district287.org](mailto:jlnelson@district287.org) 763-550-7241

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What does it mean for a student to be  
mathematically powerful?



## KRYPTO for older students



Draw 5 numbers from a half deck of cards.  
 Draw a 6<sup>th</sup> number for the target number.  
 Students create equations and record on table posters or contribute to a poster on the board.

Try:

<http://mphgames.com>



WARNING: It could be impossible to make the target number!

You can use 2 of each digit, 1-10 from a deck of cards or use a Krypto deck (numbers 1-25 with 2 of each number 1-10) or use a number deck. Half a deck may be more manageable but you can try a full deck, too!

<http://illuminations.nctm.org>

Go to activities tab, search for Krypto.

2 9 7 4 5 Target 3

Deal  
 Hint  
 Solve

Primary Krypto uses just the digits 1-10.

### What do each of these equal on your calculator?

$3 + 4 \times 5 = ?$  35 or 23?

$10 - 6 + 5 = ?$  -1 or 9?

Parenthesis & Exponents

PE Please Excuse

Mult & Divide (in order)

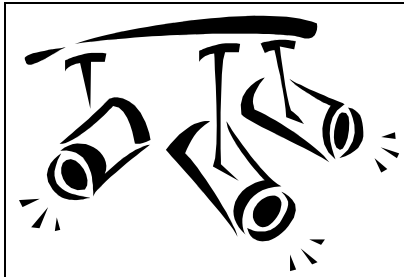
MD My Dear

Add & Subtract (in order)

AS Aunt Sally

*Not all calculators operate the same. Arithmetic order vs. order of operations. Test your calculator with  $3 + 4 \times 5$ . If you get 23 your calculator follows the order of operations.*





## Spotlight on Research

$$8 + 4 = \square + 5$$

What would students say belongs in the box?

What does belong in the box?

	<b>7</b>	<b>12</b>	<b>17</b>	<b>12 &amp; 17</b>	<b>other</b>
Gr. 1 & 2					
Gr. 3 & 4					
Gr. 5 & 6					

NCISLA inBrief: "Building a Foundation for Learning Algebra," Fall 2000,

<http://ncisla.wceruw.org/publications/briefs/fall2000.pdf>.

Carpenter, Franke and Levi. (2003). *Thinking Mathematically: Integrating Arithmetic and Algebra in Elementary School*. Heinemann: Portsmouth, NH [www.heinemann.com](http://www.heinemann.com)

Falkner, Levi and Carpenter. "Children's Understanding of Equality: A Foundation for Algebra." *Teaching Children Mathematics*. December 1999. [www.NCTM.org](http://www.NCTM.org)

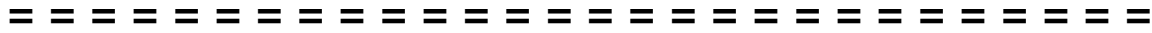
What if the problem were presented as:

$$8 + \square = 7 + 5 \quad \text{or} \quad 8 + 4 = 7 + \square$$

$$8 + 4 = k + 5 \quad \text{or} \quad 8 + 4 = 7 + n$$

# Benchmarks in Student Thinking

## About The Equal Sign



### 1. BASIC NUMBER SENTENCE SENSE

Children begin to write number sentences and describe their thinking about the equal sign. They begin to see that numbers or expressions on one side of the equal sign are *the same amount* as numbers or expressions on the other side.

### 2. EXPERIENCE WITH A VARIETY OF EQUATION TYPES

Children accept as true number sentences that go beyond the form  $a + b = c$ . They understand that equations in these forms might be true:

$$\begin{aligned}7 &= 3 + 4 \\2 + 8 &= 5 + 5 \\356 + 42 &= 354 + 44\end{aligned}$$

### 3. CALCULATING TO DETERMINE TRUTH (Operational Thinking)

Children recognize that the equal sign separates two equal values. They carry out calculations to determine that the two sides of an equation are equal or not equal.

$$\begin{array}{ccc}8 + 4 = & \underline{\quad} + 5 \\ \checkmark & \checkmark \\ 12 & 12\end{array}$$

Children need to calculate both sides to fill in the missing value.

### 4. RELATIONAL THINKING

Children compare the expressions on each side of the equation and check for truth by identifying relationships among numbers and reasoning instead of actually carrying out the calculations.

$$8 + 4 = \underline{\quad} + 5$$

*“7 is the missing number because 5 is one more than 4, so I need a number that is one less than 8.”*

Adapted from: Carprenter, Franke and Levi. *Thinking Mathematically: Integrating Arithmetic and Algebra in Elementary School*. Heinemann. Portsmouth, NH 2003 [www.heinemann.com](http://www.heinemann.com)



Video Snapshot  
(DVD 1.1) (CD 2.1)  
1'40"

Kevin – Kgn.

Equation given to student:

$$4 + 5 = \square + 3$$

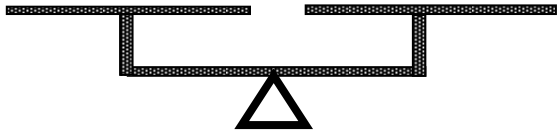
What does Kevin understand about the equal sign?

If you were this child's teacher, what problems would you have him work on next?

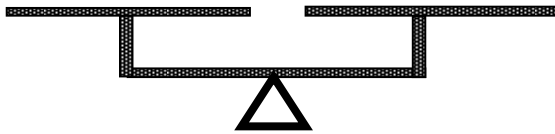


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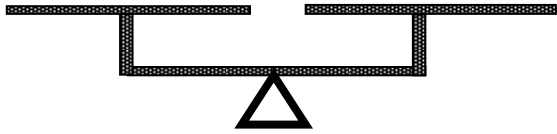
$$4 = \square$$



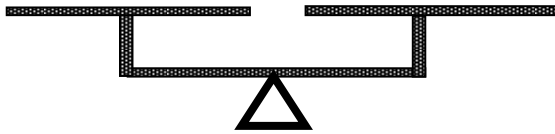
$$7 = \square$$



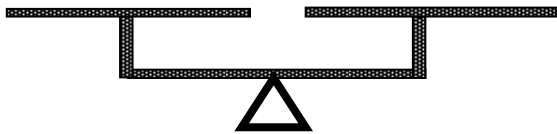
$$2 = \square$$



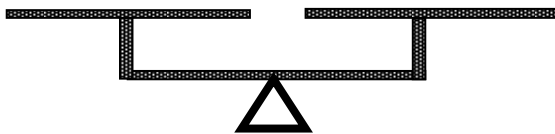
$$5 = \square$$



$$3 = \square$$



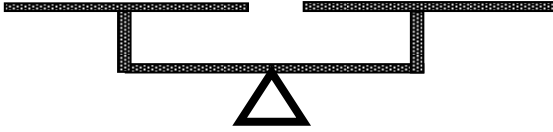
$$10 = \square$$



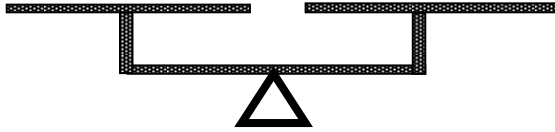
What do you notice?

Name \_\_\_\_\_ Date \_\_\_\_\_

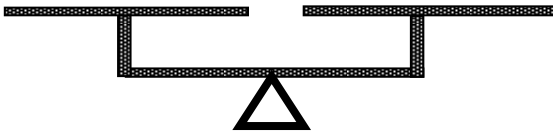
$$10 = 1 + \square$$



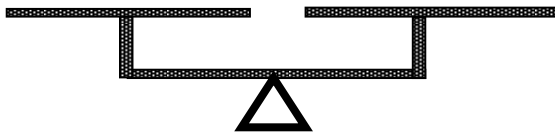
$$10 = 9 + \square$$



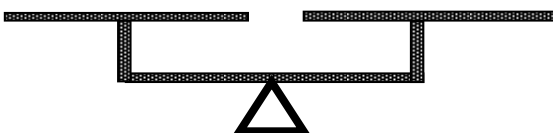
$$10 = 3 + \square$$



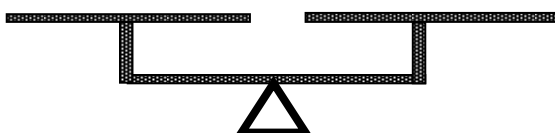
$$10 = 7 + \square$$



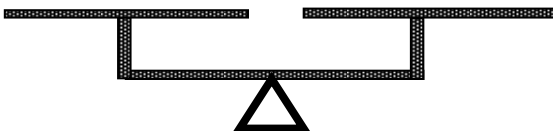
$$10 = 2 + \square$$



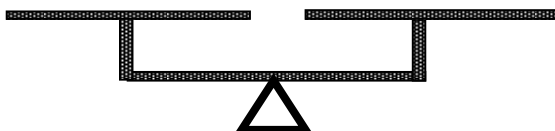
$$10 = 8 + \square$$



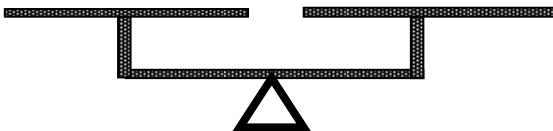
$$10 = 4 + \square$$



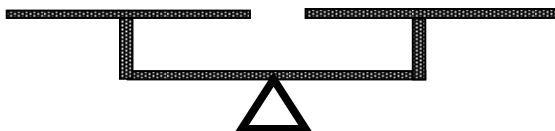
$$10 = 6 + \square$$



$$10 = 5 + \square$$





$$10 = 5 + \square$$





What do you notice?


Name \_\_\_\_\_ Date \_\_\_\_\_


$$14 + 16 = 13 + \square$$



$$14 + 16 = 12 + \square$$



$$14 + 16 = 17 + \square$$



$$14 + 16 = 18 + \square$$



$$14 + 16 = 16 + \square$$


$$14 + 16 = 15 + \square$$


$$14 + 16 = 13 + \square$$



$$14 + 16 = 12 + \square$$



$$14 + 16 = 22 + \square$$



$$14 + 16 = 24 + \square$$



What did you notice?


Name \_\_\_\_\_ Date \_\_\_\_\_


$$34 + \square = 13 + 13 + 13$$



$$33 + \square = 13 + 13 + 13$$



$$32 + \square = 13 + 13 + 13$$



$$31 + \square = 13 + 13 + 13$$



$$32 + \square = 23 + 23 + 23$$


$$31 + \square = 23 + 23 + 23$$


$$62 + \square = 23 + 23 + 23$$



$$61 + \square = 23 + 23 + 23$$



$$47 + \square = 33 + 33 + 33$$



$$45 + \square = 33 + 33 + 33$$



What did you notice?

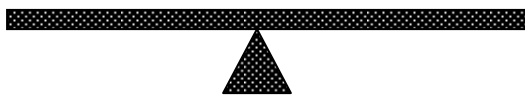
Name \_\_\_\_\_ Date \_\_\_\_\_


$$\frac{1}{2} + \frac{3}{8} = \frac{5}{8} + \boxed{\phantom{00}}$$



$$\frac{1}{2} + \frac{3}{8} = \frac{4}{8} + \boxed{\phantom{00}}$$



$$\frac{1}{2} + \frac{3}{8} = \frac{6}{8} + \boxed{\phantom{00}}$$


$$\frac{1}{2} + \frac{3}{8} = \frac{7}{8} + \boxed{\phantom{00}}$$


$$\frac{2}{3} + \frac{2}{3} = \frac{1}{3} + \boxed{\phantom{00}}$$


$$\frac{2}{3} + \frac{2}{3} = \frac{1}{6} + \boxed{\phantom{00}}$$


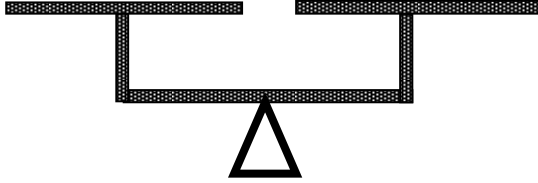
$$\frac{2}{3} + \frac{2}{3} = \frac{2}{6} + \boxed{\phantom{00}}$$


$$\frac{2}{3} + \frac{2}{3} = \frac{5}{6} + \boxed{\phantom{00}}$$


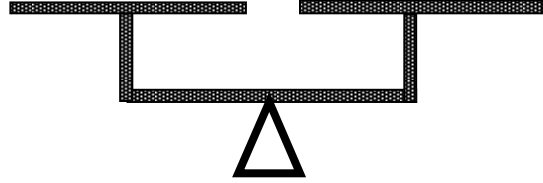
What did you notice?

Name \_\_\_\_\_ Date \_\_\_\_\_

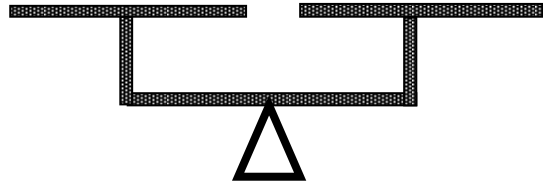
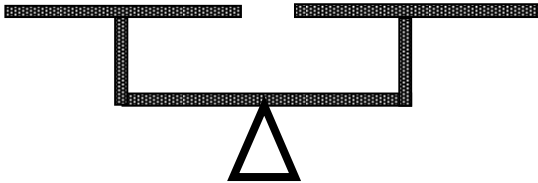
$$4 \cdot 3 = 3 \cdot n$$



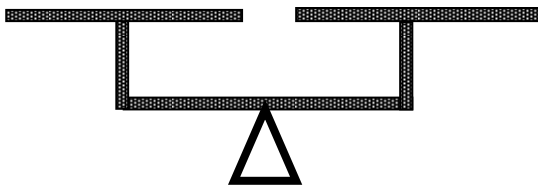
$$5 \cdot 6 = 6 \cdot n$$



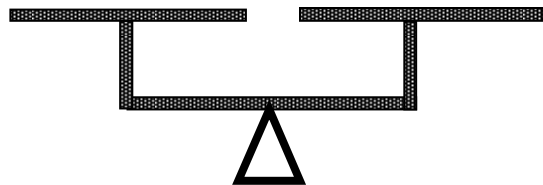
Prove each of the above equations with a drawing below.



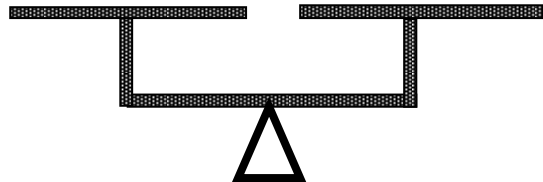
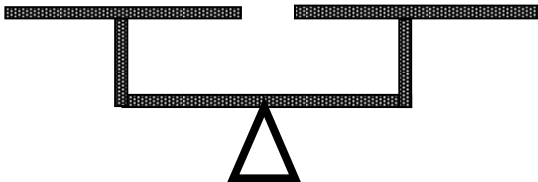
$$4 \cdot 5 = 5 \cdot n$$



$$3 \cdot 5 = 5 \cdot n$$



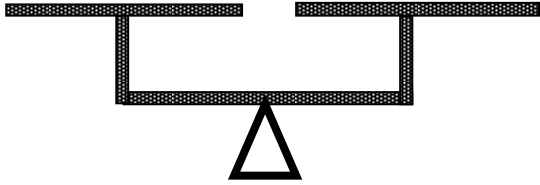
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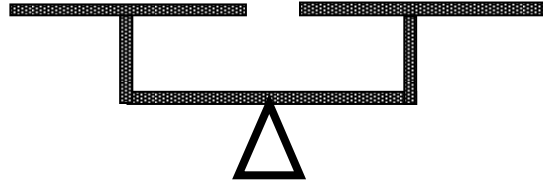
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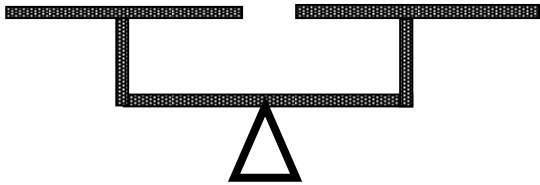
$$8x + 7 = 5x + \underline{\quad} + 7$$



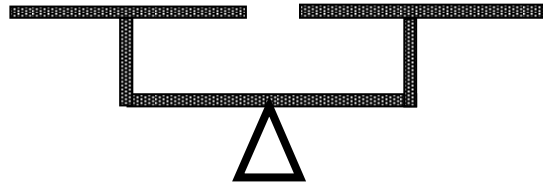
$$8x + 7 = 8x + \underline{\quad} + 3$$



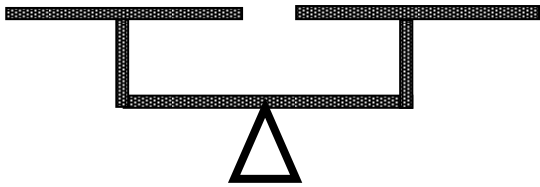
$$7x + 7 = 6x + \underline{\quad} + 7$$



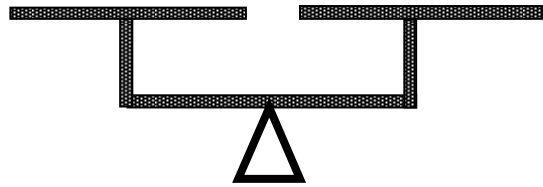
$$7x + 7 = 7x + \underline{\quad} + 6$$



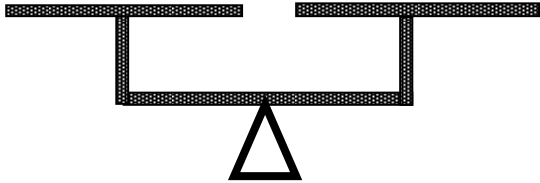
$$5x + 12 = 3x + \underline{\quad} + 7 + \underline{\quad}$$



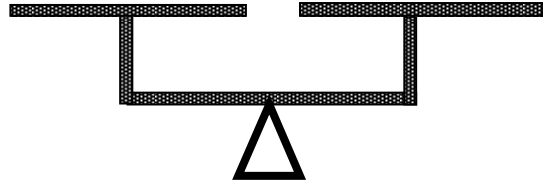
$$5x + 12 = 4x + \underline{\quad} + 9 + \underline{\quad}$$



$$12x + 12 = 3x + \underline{\quad} + 7 + \underline{\quad}$$

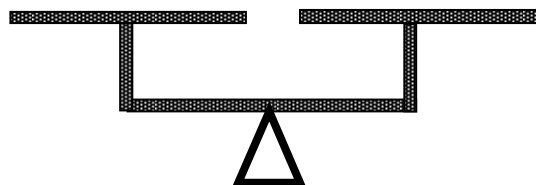
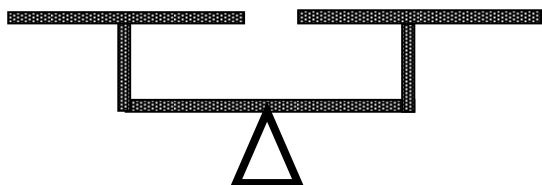
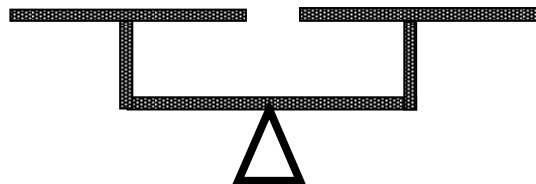
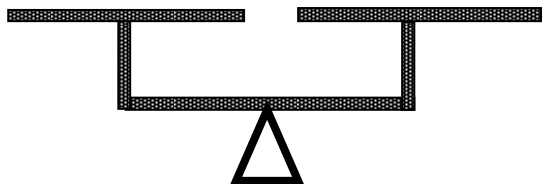
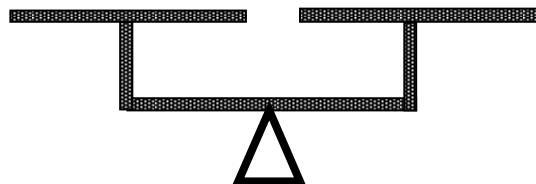
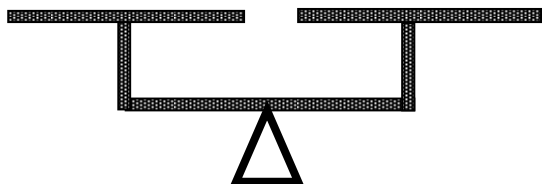
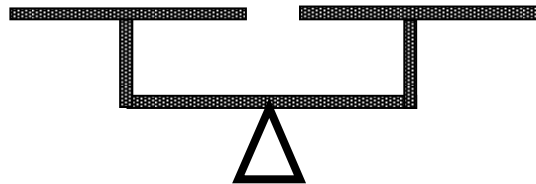
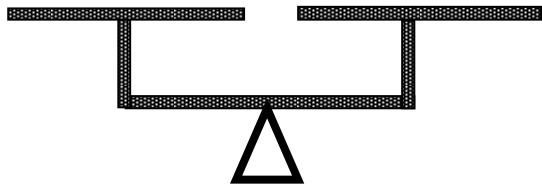


$$12x + 12 = 4x + \underline{\quad} + 9 + \underline{\quad}$$



What did you notice?

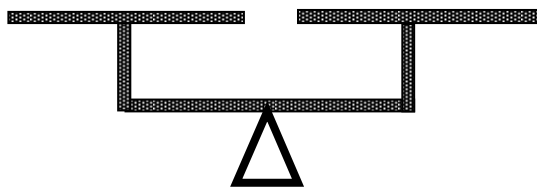
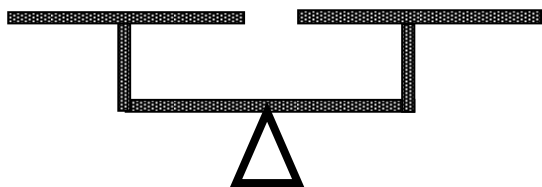
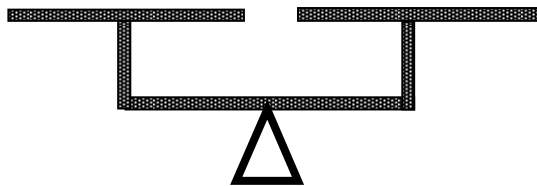
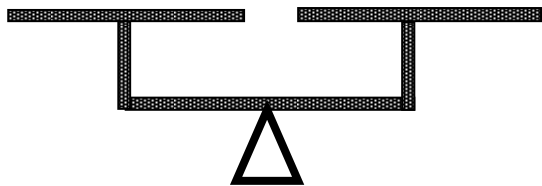
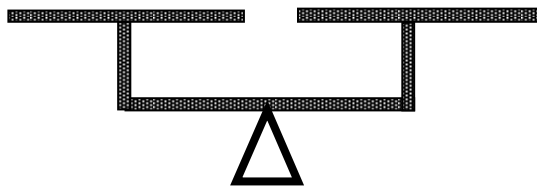
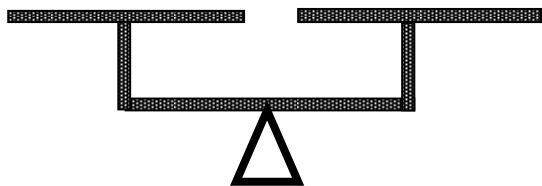
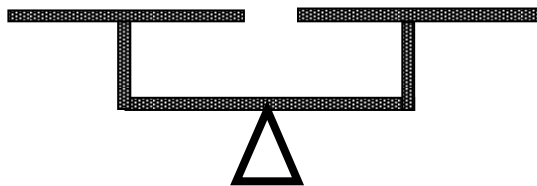
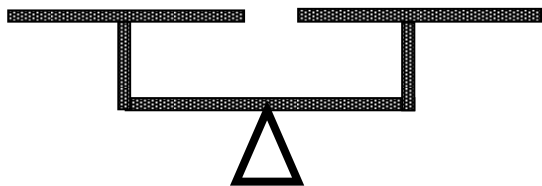
Name \_\_\_\_\_ Date \_\_\_\_\_



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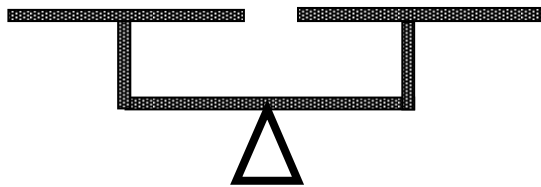
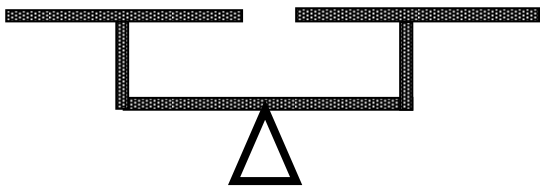
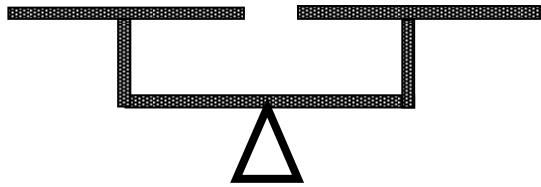
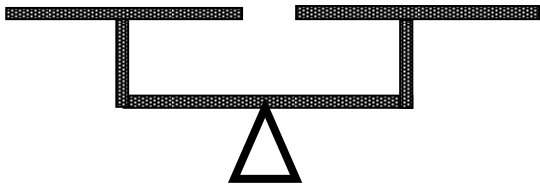
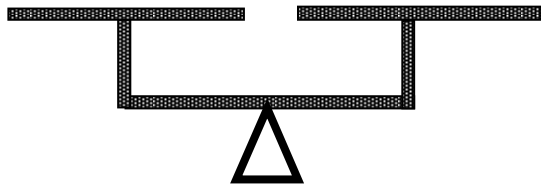
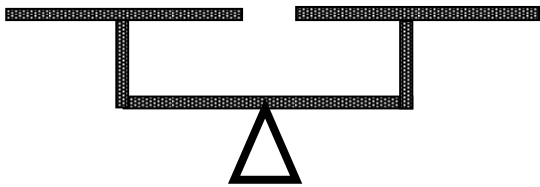
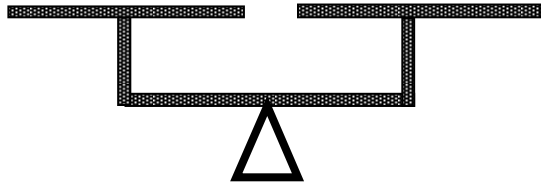
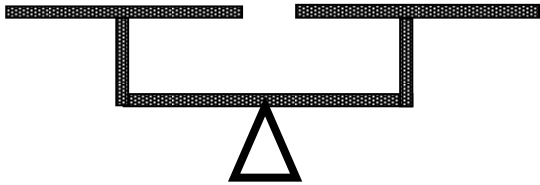


Name \_\_\_\_\_ Date \_\_\_\_\_



What did you notice?

Name \_\_\_\_\_ Date \_\_\_\_\_



What did you notice?



Video Snapshot  
(DVD 1.3) (CD 2.3)  
10'19"

## 4<sup>th</sup> Grade Bilingual Classroom

Equations given to students:

$$8 + 4 = \square + 5$$

$$7 = 3 + 4$$

$$6 = 6 + 0$$

$$6 = 6$$

$$5 = 5$$

$$5 = 4 + 1$$

$$6 = 3 + 3$$

$$15 + 4 = \square + 11$$

Why is each number sentence useful in developing students thinking about equality?

If you were these children's teacher, what equations might you use?

## Some comments on the equal sign . . .

What is wrong with this notation?

$$8 + 4 = 12 + 5 = 17$$



A **“goes to arrow”** is used to track ongoing thinking.

$$8 + 4 \rightarrow 12 + 5 \rightarrow 17$$

An **“equation chain”** can use multiple equal signs if the expressions surrounding any equal sign are equal to each other. For example, children might generate many ways to make 10 and write the following “equation chain”:

$$10 = 6 + 4 = 7 + 3 = 20 - 10 = 100 - 90 = 7 + 2 + 1$$

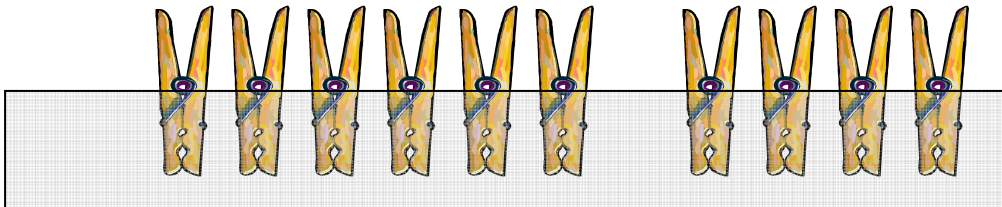
*Consider having students create chains on adding machine tape to encourage flexible thinking about a given quantity and expressions that represent that amount.*

A yellow ribbon with a black outline and a black shadow, resembling a piece of adding machine tape. The ribbon is horizontal and has a pointed end on the left and a pointed end on the right. Inside the ribbon, the equation chain  $75 = 3 \times 25 = 100 - 25 = 7 \times 10 + 5$  is written in black text. The numbers and symbols are centered within the ribbon's width.

## INTRODUCING TRUE OR FALSE WORK WITH EQUATIONS

It is difficult to sort out exactly why misconceptions about the meaning of the equal sign are so pervasive and so persistent. A good guess is that many children see only examples of number sentences with an operation to the left of the equal sign and the answer on the right and they over generalize from those limited examples.

Carpenter, Franke and Levi. *Thinking Mathematically: Integrating Arithmetic and Algebra in Elementary School*. Heinemann. Portsmouth, NH 2003 [www.heinemann.com](http://www.heinemann.com) p. 22



### The Clothespin Card

$6 + 4 = 10$	$0 + 10 = 10$	$10 = 6 + 4$
$4 + 6 = 10$	$1 + 9 = 10$	$10 = 4 + 6$
	$2 + 8 = 10$	
$7 + 3 = 10$	$3 + 7 = 10$	$10 = 7 + 3$
$3 + 7 = 10$	$4 + 6 = 10$	$10 = 3 + 7$
	$5 + 5 = 10$	
$5 + 5 = 10$	$6 + 4 = 10$	$10 + 5 + 5$
	$7 + 3 = 10$	
$2 + 8 = 10$	$8 + 2 = 10$	$10 = 2 + 8$
$8 + 2 = 10$	$9 + 1 = 10$	$10 = 8 + 2$
	$10 + 0 = 10$	
$9 + 1 = 10$		$10 = 9 + 1$
$1 + 9 = 10$		$10 = 1 + 9$
$10 + 0 = 10$		$10 = 10 + 0$
$0 + 10 = 10$		$10 = 0 + 10$

- ✓ Algebrafy basic facts by the questions you ask: e.g. *How do you know you have found all the combinations?*
- ✓ Use true/false equations to tease out different formats for equations
- ✓ ***“Math people can do things forwards and backwards – as long as they tell the truth!”***



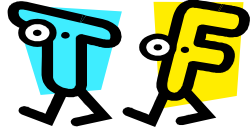
# True or False?

Order from Easiest to Hardest for Student Thinking



T or F? $3 + 8 = 10$	T or F? $73 + 56 = 71 + 54$
T or F? $10 = 3 + 7$	T or F? $68 + 58 = 57 + 69$
T or F? $5 + 5 = 3 + 7$	T or F? $37 + 56 = 37 + 58$
T or F? $1 + 4 + 5 = 5 + 5$	T or F? $87 + 45 = 88 + 46$
T or F? $10 = 3 + 5$	T or F? $234 + 578 = 234 + 578 + 8$
T or F? $10 = 5 + 3 + 3$	T or F? $94 + 87 - 38 = 94 + 85 - 36$
T or F? $10 = 1 + 5 + 4$	T or F? $59 + 63 = 69 + 53$
T or F? $37 \times 54 = 35 \times 54 + 2 \times 54$	T or F? $2 \frac{1}{2} + \frac{3}{4} = 3 \frac{1}{4}$
T or F? $20 \times 54 = 40 \times 108$	T or F? $5 \frac{3}{4} + \frac{1}{2} = 5 \frac{3}{4}$
T or F? $5 \times 48 = 10 \times 24$	T or F? $12 \frac{3}{4} + 2 \frac{1}{4} = 15 \frac{1}{4}$
T or F? $64 \div 14 = 32 \div 7$	T or F? $6 \frac{1}{2} + \frac{3}{4} = 7$
T or F? $42 \div 32 = 84 \div 32$	T or F? $\frac{3}{4} + \frac{1}{2} = 1 \frac{1}{4}$
T or F? $48 \div 24 = 24 \div 12$	T or F? $10 = 5 \frac{1}{2} + 5 \frac{1}{2}$
T or F? $40 \times 60 = 480 \times 6$	T or F? $1 \frac{1}{4} + \frac{1}{4} = 1 \frac{1}{2}$

Adapted from: Carpenter, Franke and Levi. *Thinking Mathematically: Integrating Arithmetic and Algebra in Elementary School*. Heinemann. Portsmouth, NH 2003 [www.heinemann.com](http://www.heinemann.com)



A Week's Worth of  
True or False or Open Number Equations  
for My Students

Monday	
Tuesday	
Wednesday	
Thursday	
Friday	

# Which Symbol Makes the Sentence True?

$=$   
equal

$\neq$   
not equal

Sample page  
for adult  
learners

	$=$ or $\neq$	
6		6
5		6
13		31
27		72
1,329		1329
$\frac{1}{2}$		$\frac{3}{6}$
.75		$\frac{4}{5}$
363		$36 \cdot 10 + 3$
363		$3 \cdot 100 + 36 \cdot 10 + 3$
3,063		$3 \cdot 100 + 6 \cdot 100 + 3$

What did you notice?



Name \_\_\_\_\_ Date \_\_\_\_\_

### Which Symbol Makes the Sentence True?

$=$  or  $\neq$   
equal or not equal

	$=$ or $\neq$	
$4 + 1$		5
$4 + 2$		5
$6 + 1$		7
$3 + 1$		4
$3 + 2$		4
$2 + 1$		3
$5 + 1$		6

What did you notice?

Name \_\_\_\_\_ Date \_\_\_\_\_

## Which Symbol Makes the Sentence True?

$=$  or  $\neq$   
equal or not equal

	$=$ or $\neq$	
6		$5 + 1$
6		$5 + 2$
4		$3 + 1$
3		$2 + 1$
5		$4 + 1$
5		$4 + 2$
7		$6 + 1$

What did you notice?

Name \_\_\_\_\_ Date \_\_\_\_\_

### Which Symbol Makes the Sentence True?

$=$  or  $\neq$   
equal or not equal

	$=$ or $\neq$	
$24 + 10$		34
$22 + 10$		34
$35 + 10$		55
$45 + 10$		55
$37 + 20$		67
$47 + 20$		67
$55 + 40$		95

What did you notice?

### Which Symbol Makes the Sentence True?

$=$                        $\neq$   
 equal                      not equal

	$=$ or $\neq$	
$343 + 541$		$340 + 544$
$343 + 541$		$350 + 540$
$343 + 549$		$350 + 542$
$738 + 243$		$740 + 241$
$541 - 343$		$540 - 342$
$738 - 243$		$735 - 245$
$738 - 243$		$740 - 241$
$1003 - 697$		$1006 - 700$

What did you notice?

## Which Symbol Makes the Sentence True?

=

 $\neq$ 

equal

not equal

	= or $\neq$	
$3(4 + 6)$		$3 \cdot 4 + 3 \cdot 6$
$7(4 + 5)$		$7 \cdot 4 + 5$
$8(3 + 4)$		$8 \cdot 7$
$36 \cdot 3 + 36 \cdot 4$		$36(3 \cdot 4)$
$7 \cdot 6 \cdot 5$		$5 \cdot 6 \cdot 7$
$42 \cdot 2 \cdot 5$		$42 \cdot 25$
$42 \cdot 2 \cdot 5$		$42 \cdot 10$
$10 \cdot 66$		$5 \cdot 2 \cdot 33$

What did you notice?

Name \_\_\_\_\_ Date \_\_\_\_\_

### Which Symbol Makes the Sentence True?

$=$  or  $\neq$   
equal or not equal

	$=$ or $\neq$	
$\frac{1}{2}$		$\frac{4}{8}$
$\frac{1}{2}$		$\frac{2}{4}$
$\frac{1}{2}$		$\frac{7}{8}$
$\frac{1}{2}$		$\frac{1}{4}$
$\frac{1}{2}$		$\frac{6}{12}$
$\frac{1}{2}$		$\frac{7}{12}$
$\frac{1}{2}$		$\frac{3}{6}$

What did you notice?

Name \_\_\_\_\_ Date \_\_\_\_\_

## Which Symbol Makes the Sentence True?

$=$                        $\neq$   
equal                      not equal

	$=$ or $\neq$	

What did you notice?

Name \_\_\_\_\_ Date \_\_\_\_\_

## Which Symbol Makes the Sentence True?

$=$                        $\neq$   
equal                      not equal

	$=$ or $\neq$	

What did you notice?



Name \_\_\_\_\_ Date \_\_\_\_\_

## Which Symbol Makes the Sentence True?

$=$                        $\neq$   
equal                      not equal

	$=$ or $\neq$	

What did you notice?

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